Analysis of Penalty Due to Low-Frequency Intensity Modulation in Optical Transmission Systems

Jin-Serk Baik, Kun-Youl Park, Tae-Won Oh, and Chang-Hee Lee, *Member, IEEE, Member, OSA*

***Abstract*—The low-frequency intensity modulation in optical transmission systems can occurred by optical performance monitoring or dynamic devices such as acoustooptic tunable filters. Using a conventional receiver model, we derived the optical signal-to-noise ratio (OSNR) and Q penalties due to the low-frequency intensity modulation. The penalty is a function of the extinction ratio, OSNR, the modulation index, and the number of pilot-tone signals. The analytic results show good agreement with the experimental results.**

***Index Terms*—Bit error rate (BER), intensity modulation, optical signal-to-noise ratio (OSNR) penalty, Q penalty.**

I. INTRODUCTION

OPTICAL path monitoring and an optical signal degradation monitoring schemes had been used as pilot tones or subcarrier signals of low frequency compared to the bit rate of a line signal [1], [2]. These additional tones modulate the intensity of original optical data signals and give inevitable penalties. Furthermore, dynamic devices such as acoustooptic tunable filters (AOTFs) that are driven by signals of a few megahertz may also give a penalty to the system due to intensity modulation [3]. In order to use these monitoring schemes or devices, we should estimate the additional penalty caused by the low-frequency intensity modulations.

However, the penalty induced by low-frequency intensity modulation has not been analyzed clearly. There were a few analyses that arose from interferometric noise [4] or chirp-induced intensity fluctuation in modulator [5], which can be referred to in order to analyze the penalty by low-frequency modulation. The SNR penalty estimated by Murakami *et al.* [6] coincides with their measured data for a low-frequency small-amplitude pilot tone. However, it is limited to interference of supervisory signal in the line signal. Thus it is hard to gain insight for the case of multiple modulation tones imposed on the line signals.

In this paper, we propose a novel model for the optical receiver suffered from the undesired low-frequency intensity modulation as a function of the modulation index and the number of tones. In Section II, we calculate Q factor using the proposed model, and then show the bit error rate (BER) directly as an analytic form for a single modulation tone with a small modulation index. The resultant BER is described by the modified Bessel function of the first kind of the modulation index. We extend the single-tone results to multiple modulation tones. Then, the BER can be described as a multiplication of the modified Bessel functions for each modulation. In Sections II and III, we numerically calculate the penalty for the various modulation indexes and the number of tones and compare the results with the experimentally measured data. The theoretical results agree well with the measured results for a small modulation index. Finally, we discuss the limit of line-signal transmission due to the low-frequency intensity modulating tones.

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The authors are with the Department of Electrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology, 305-701 Daejeon, Korea (e-mail: chl@ee.kaist.ac.kr).

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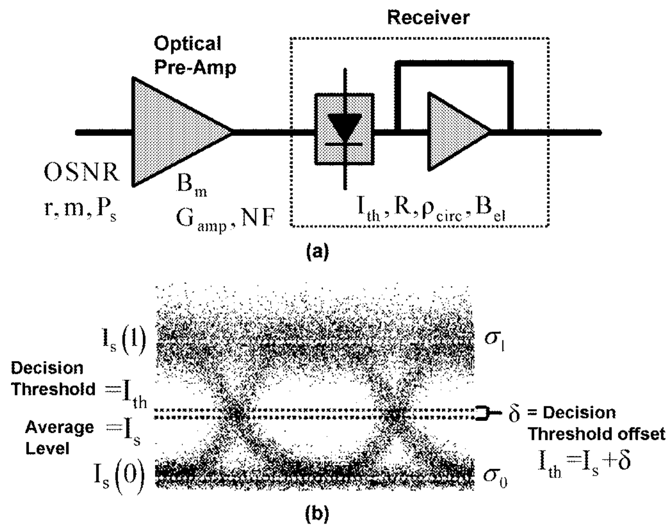


Fig. 1. (a) Receiver model used in low-frequency amplitude modulation analysis and (b) definition of notation.

II. MODELING AND DERIVATION

Fig. 1 shows an optical receiver with a typical eye-diagram. We also show the definition of notations used in modulation penalty derivation in the figure. I_s is the average output current level of the receiver corresponding to P_s. $\sigma_{0,1}$ represents the standard deviation of noise for "0" and "1" level, respectively. We adopted an optical preamplifier before the receiver with the gain G_{amp} and the noise figure NF. Other parameters [i.e., optical input power per channel, optical signal-to-noise ratio (OSNR), extinction ratio, etc.] are summarized in Table I.

A. Single-Tone Case

We begin with the case that single modulation tone is imposed on the line signal. Then, we can express the modulated signal current at the receiver output as

$$I_s(y) = RG_{\mathrm{amp}}P_s(y)\left(1 + m\cos\omega_m t\right), \ y = 0, 1$$
$$\text{where } P_s(1) = \frac{2}{1+r}P_s, \quad P_s(0) = \frac{2r}{1+r}P_s \tag{1}$$

TABLE I
NOMENCLATURE

Parameters	Description
OSNR	Optical SNR at the input of the preamplifier
P_s	Average input optical power of the preamplifier
r	Extinction ratio of the line-signal
m	Intensity modulation index of the low frequency tone
G_{amp}	Gain of the preamplifier
NF	Noise figure of the preamplifier
B_m	Optical bandwidth of the channel (dependent on demux bandwidth such as AWG or OADM)
I_{th}	Decision threshold of the receiver
R	Responsivity of the receiver
ρ_{circ}	Electrical noise density in the receiver
B_{el}	Electrical bandwidth of the receiver

where $P_s(y)$ is the optical signal power entering into the receiver and ω_m is the modulation frequency. As shown in Fig. 1(b), we considered another parameter δ, which is the offset of the decision threshold from its average level. Then we get $I_{th} = I_s + \delta$, where I_s represents the average value of $I_s(1)$ and $I_s(0)$. We define $\tilde{I}$, which is given by

$$\begin{aligned}\tilde{I}_s(1) &\equiv I_s(1) - I_s \\ \tilde{I}_s(0) &\equiv I_s(0) - I_s.\end{aligned} \tag{2}$$

The noise current due to the ASE with two polarizations can be expressed as the sum of input ASE and the additional ASE generated in preamplifier. Then we have

$$\begin{aligned}I_N &= RP_N G_{\rm amp} + 2Rhv n_{sp}\left(G_{\rm amp}-1\right)B_m \\ &= R\frac{G_{\rm amp}P_s}{OSNR\frac{B_o}{B_m}} + 2Rhv n_{sp}\left(G_{\rm amp}-1\right)B_m\end{aligned} \tag{3}$$

where P_N represents the noise power from the preamplifier input, hv is the photon energy, n_{sp} is the spontaneous emission factor, and B_0 is the resolution bandwidth to get the noise power level. Then, we can describe the detected noise as

$$\begin{aligned}N(y) = I_{circ} + 2B_{el}\left(e+\frac{I_N}{B_m}\right)I_s(y) \\ +2B_{el}I_N\left(e+\frac{I_N}{2B_m}\right), \quad y=0,1\end{aligned} \tag{4}$$

where the first term represents the circuit noise power that can be obtained as $\rho_{circ}^2 B_{el}$, the second term represents the signal shot noise plus the signal amplified spontaneous emission (ASE) beating, and the third term represents the ASE shot noise plus the ASE–ASE beating. The standard deviations of noise for "1" level and "0" level are given by

$$\sigma_1 = \sqrt{N(1)}, \quad \sigma_0 = \sqrt{N(0)}. \tag{5}$$

The Q-factors for "1" level Q_1 and "0" level Q_0 are given by

$$Q_1 = \frac{\tilde{I}_s(1)-\delta}{\sigma_1}, \quad Q_0 = \frac{\delta - \tilde{I}_s(0)}{\sigma_0}. \tag{6}$$

Substituting (2) and (5) into (6), we have Q_1 as shown in (7) at the bottom of the page.

Here, the denominator of the (7) can be simplified using the binomial approximation. For a small modulation index ($m \ll 1$), we have

$$\begin{aligned}&\{A(1+m\cos\omega_m t)+B\}^{-1/2} \\ &= (A+B)^{-1/2}\left\{1+\frac{A}{A+B}m\cos\omega_m t\right\}^{-1/2} \\ &\cong \frac{1-\frac{1}{2}\frac{A}{A+B}m\cos\omega_m t}{(A+B)^{1/2}}.\end{aligned} \tag{8}$$

Thus, Q_1 can be expressed as shown in (9) at the bottom of the page.

$$\begin{aligned}Q_1 &= \frac{RG_{\rm amp}P_s\frac{1-r}{1+r}\left(1+m\cos\omega_m t\right)-\delta}{\left\{\frac{4}{1+r}RG_{\rm amp}P_sB_{el}\left(e+\frac{I_N}{B_m}\right)\left(1+m\cos\omega_m t\right)+I_{circ}+2B_{el}I_N\left(e+\frac{I_N}{2B_m}\right)\right\}^{1/2}} \\ &= \frac{RG_{\rm amp}P_s\frac{1-r}{1+r}\left(1+m\cos\omega_m t\right)-\delta}{\left\{A\left(1+m\cos\omega_m t\right)+B\right\}^{1/2}} \\ \text{where } A &= \frac{4}{1+r}RG_{\rm amp}P_sB_{el}\left(e+\frac{I_N}{B_m}\right) \\ B &= I_{circ}+2B_{el}I_N\left(e+\frac{I_N}{2B_m}\right)\end{aligned} \tag{7}$$

$$Q_1 \cong \frac{RG_{\rm amp}P_s\frac{1-r}{1+r}-\delta+\left\{RG_{\rm amp}P_s\frac{1-r}{1+r}\left(1-\frac{1}{2}\frac{A}{A+B}\right)+\frac{\delta}{2}\frac{A}{A+B}\right\}m\cos\omega_m t}{(A+B)^{1/2}} \tag{9}$$

Similarly, we can obtain Q_0 as shown in (10) at the bottom of the page, where A and B are defined in (7).

Typically the signal-ASE beating is much higher than the thermal noise, the ASE shot noise, and the ASE–ASE beating noise ($A \gg B$). If we assume the ideal case of infinite extinction ratio, i.e., $r=0$, we can simplify (9) and (10) as

$$Q_1 \cong \frac{RG_{\rm amp}P_s - \delta + \frac{1}{2}\left\{RG_{\rm amp}P_s + \delta\right\} m\cos\omega_m t}{\sqrt{A}}$$
$$Q_0 \cong \frac{RG_{\rm amp}P_s + \delta + RG_{\rm amp}P_s m\cos\omega_m t}{\sqrt{B}}. \quad (11)$$

Since we have Q_1 and Q_0, we can obtain BER. If levels "1" and "0" have the same probability of being transmitted, the BER is given by

$$\mathrm{BER} = \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}\alpha_1}\exp\left(-\frac{\alpha_1^2}{2}\right)\mathrm{I}_0\left(\alpha_1^2\beta_1 m\right) + \frac{1}{\sqrt{2\pi}\alpha_0}\exp\left(-\frac{\alpha_0^2}{2}\right)\mathrm{I}_0\left(\alpha_0^2\beta_0 m\right)\right] \quad (12)$$

where the definitions of the coefficients α_i and β_i are given in the Appendix . See the Appendix also for derivation of (12).

B. ASE Modulation Effect

In an optical transmission system with many optical amplifiers, the ASE is also modulated by the low frequencies if we use the acoustooptic filters at the midstage of each amplifier for the gain equalization. Here, we assume the same modulation index for the ASE. Then, we can rewrite the σ_1 and σ_0 in (4) with the following modulation related ASE modulation:

$$I'_N = I_N\left(1 + m\cos\omega_m t\right)$$

where

$$I_N = R\frac{G_{\rm amp}P_s}{\mathrm{OSNR}\frac{B_0}{B_m}} + 2Rh\nu n_{sp}\left(G_{\rm amp}-1\right)B_m. \quad (13)$$

Replacing I_N by I'_N in (4) and using (5), we can get each new noise variance value for the "0" level and "1" level under the ASE modulation. We also assume a small modulation index $m \ll 1$. See (14) at the bottom of the page.

Using (6) and (14), we obtain the following Q factors. Here, we neglect the terms of m^2 and the higher order terms in binomial approximation

$$Q_1 = \frac{RG_{\rm amp}P_s\frac{1-r}{1+r} - \delta}{\sqrt{C_1}} \times\left[1 + \frac{RG_{\rm amp}P_s\left(\frac{2}{1+r} - \frac{1-r}{1+r}\frac{D_1}{2C_1}\right) + \frac{\delta D_1}{2C_1}}{RG_{\rm amp}P_s\frac{1-r}{1+r} - \delta} m\cos\omega_m t\right]$$
$$Q_0 = \frac{RG_{\rm amp}P_s\frac{1-r}{1+r} + \delta}{\sqrt{C_0}} \times\left[1 - \frac{RG_{\rm amp}P_s\left(\frac{2r}{1+r} - \frac{1-r}{1+r}\frac{D_0}{2C_0}\right) + \frac{\delta D_0}{2C_0}}{RG_{\rm amp}P_s\frac{1-r}{1+r} + \delta} m\cos\omega_m t\right]. \quad (15)$$

If the signal shot noise is dominant, $C_i \cong D_i$. Assuming $r=0$, we have

$$Q_1 = \frac{RG_{\rm amp}P_s - \delta}{\sqrt{C_1}}\left[1 + \frac{\frac{3}{2}RG_{\rm amp}P_s + \frac{\delta}{2}}{RG_{\rm amp}P_s - \delta} m\cos\omega_m t\right]$$
$$Q_0 = \frac{RG_{\rm amp}P_s + \delta}{\sqrt{C_0}}\left[1 + \frac{\frac{1}{2}RG_{\rm amp}P_s - \frac{\delta}{2}}{RG_{\rm amp}P_s + \delta} m\cos\omega_m t\right]. \quad (16)$$

Once again, we can define appropriate coefficients α_i and β_i in (15) corresponding to (A1), and BER can be defined the same as (12) in the optical signal with the modulated ASE.

C. Extension of Model to Multitone Case

In an optical transmission system with multiple low-frequency tones for the optical performance monitoring, we have

$$Q_0 \cong \frac{RG_{\rm amp}P_s\frac{1-r}{1+r} + \delta + \left\{RG_{\rm amp}P_s\frac{1-r}{1+r}\left(1 - \frac{1}{2}\frac{Ar}{Ar+B}\right) - \frac{\delta}{2}\frac{Ar}{Ar+B}\right\} m\cos\omega_m t}{(Ar+B)^{1/2}} \quad (10)$$

$$\sigma_1^2 = C_1\left[1 + \frac{D_1}{C_1} m\cos\omega_m t\right]$$
$$\sigma_0^2 = C_0\left[1 + \frac{D_0}{C_0} m\cos\omega_m t\right]$$
$$\text{where}\begin{cases} C_1 = 2B_{el}e\left[RG_{\rm amp}\frac{2}{1+r}P_s + I_N\right] + 2B_{el}\frac{I_N}{B_m}\left[RG_{\rm amp}\frac{2}{1+r}P_s + \frac{I_N}{2}\right] + I_{circ} \\ D_1 = 2B_{el}e\left[RG_{\rm amp}\frac{2}{1+r}P_s + I_N\right] + 4B_{el}\frac{I_N}{B_m}\left[RG_{\rm amp}\frac{2}{1+r}P_s + \frac{I_N}{2}\right] \\ C_0 = 2B_{el}e\left[RG_{\rm amp}\frac{2r}{1+r}P_s + I_N\right] + 2B_{el}\frac{I_N}{B_m}\left[RG_{\rm amp}\frac{2r}{1+r}P_s + \frac{I_N}{2}\right] + I_{circ} \\ D_0 = 2B_{el}e\left[RG_{\rm amp}\frac{2r}{1+r}P_s + I_N\right] + 4B_{el}\frac{I_N}{B_m}\left[RG_{\rm amp}\frac{2r}{1+r}P_s + \frac{I_N}{2}\right] \end{cases} \quad (14)$$

the multiple modulation tones on the transmitted signal. In addition, if we apply multiple low-frequency driving tones to optical active devices, we also have multiple modulation tones.

If we neglect all beating terms among the modulation tones for sufficiently small m_n, the output power with N modulation tones can be written as

$$\begin{aligned} I_s =& RG_{\mathrm{amp}}P_s \prod_{i=1}^{N}(1+m_i\cos\omega_{m_i}t) \\ \cong& RG_{\mathrm{amp}}P_s \left(1+\sum_{n=1}^{N} m_n\cos\omega_{m_n}t\right). \end{aligned} \tag{17}$$

Thus, the Q factor of the line signal can be given by

$$Q_i = \alpha_i \cdot \left(1+\beta_i\cdot\sum_{n=1}^{N} m_n\cos\omega_{m_n}t\right), i=0,1 \tag{18}$$

where α_i and β_i are a function of δ. Here, we should notice that δ is also a function of the modulation indexes.

Let T be the period of a common multiple frequency of $\omega_{m1}, \omega_{m2}, \ldots$. Then we can see

$$\begin{aligned} &\frac{1}{T}\int_0^T \frac{1}{Q_i\sqrt{2\pi}}\exp\left(-\frac{Q_i^2}{2}\right)dt \\ &\cong \frac{1}{\alpha_i\sqrt{2\pi}}\exp\left(-\frac{\alpha_i^2}{2}\right) \\ &\quad\cdot\frac{1}{T}\int_0^T\left(1-\beta_i\cdot\sum_{n=1}^{N} m_n\cos\omega_{m_n}t\right)\exp\left(-\alpha_i^2\beta_i\cdot\sum_{n=1}^{N} m_n\cos\omega_{m_n}t\right)dt \\ &\cong \frac{1}{\alpha_i\sqrt{2\pi}}\exp\left(-\frac{\alpha_i^2}{2}\right) \\ &\quad\cdot\frac{1}{T}\int_0^T \exp\left(-\alpha_i^2\beta_i\cdot\sum_{n=1}^{N} m_n\cos\omega_{m_n}t\right)dt \end{aligned} \tag{19}$$

where we neglect all the terms of m_n^2 and higher order terms. Thus, we have

$$\begin{aligned} \mathrm{BER} =& \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}\alpha_1}\exp\left(-\frac{\alpha_1^2}{2}\right)\right. \\ &\cdot\prod_{n=1}^{N} \mathrm{I}_0\left(\alpha_1^2\beta_1\cdot m_n\right) \\ &\left.+\frac{1}{\sqrt{2\pi}\alpha_0}\exp\left(-\frac{\alpha_0^2}{2}\right)\cdot\prod_{n=1}^{N}\mathrm{I}_0\left(\alpha_0^2\beta_0\cdot m_n\right)\right]. \end{aligned} \tag{20}$$

Therefore, the BER in the multitone case can be expressed as the multiplication of the modified Bessel function that was derived for a single modulation tone at a given modulation index m_n.

III. Simulation Results

We simulated the Q and OSNR penalties under the several conditions to see the influence of the low-frequency modulation.

TABLE II
Simulation Parameters

Parameters	Values
OSNR	20 ~ 35 dB
P_{in}	-19 dBm
r	9 ~ 11 dB
m	0 ~ 10 %
G_{amp}	15 dB
NF	6 dB (typical)
B_m	60 GHz (~ 0.5 nm)
I_{th}	Variable
R	~ 0.8 A/W
ρ_{circ}	14 pA/(Hz)$^{-1/2}$
B_{el}	9 GHz

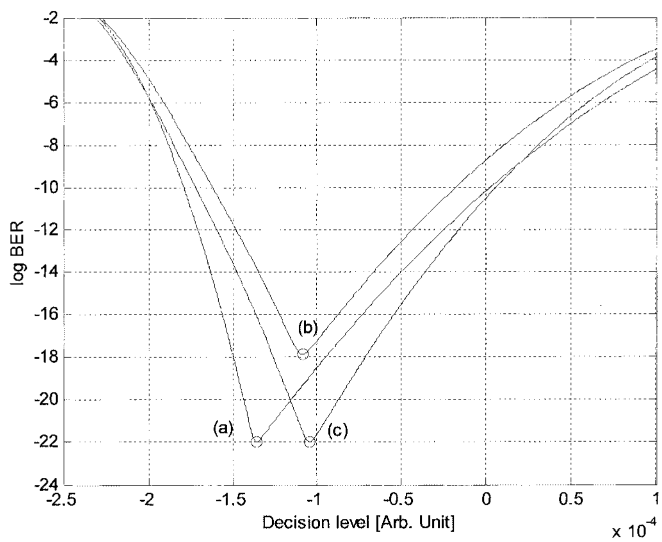


Fig. 2. V-curves that are log BER versus decision level of receiver (a) when no additional modulation tone is applied with 11 dB extinction ratio, 22-dB OSNR, and -19 dBm/ch input power to the preamplifier of 15-dB gain, (b) when eight tones with 2% modulation index are applied with no other condition changed, and (c) when OSNR is increased to 23.3 dB and the other conditions are the same as o those of (b). Here, the OSNR penalty for $8 \times 2\%$ modulation tones becomes 1.3 dB. The circle on the each curve is the minimum BER point.

We used parameters for a typical 10-Gb/s transmission system. These values are shown in Table II.

At first, we plot V-curves versus the decision threshold of the receiver by varying δ to obtain the minimum BER as shown in Fig. 2 [7]. From the curve, we can calculate the minimum obtainable BER. When we apply the modulation tones, the corresponding V-curve is shifted upward, which means that the minimum obtainable BER is also increased. By increasing the OSNR, we can restore the BER even if the modulation tones exist. Here, we define OSNR penalty due to the modulation tones as the increment of the OSNR to get the same BER.

In Fig. 3, we plot the required OSNR versus the modulation index variation with two different extinction ratios at 10^{-16}

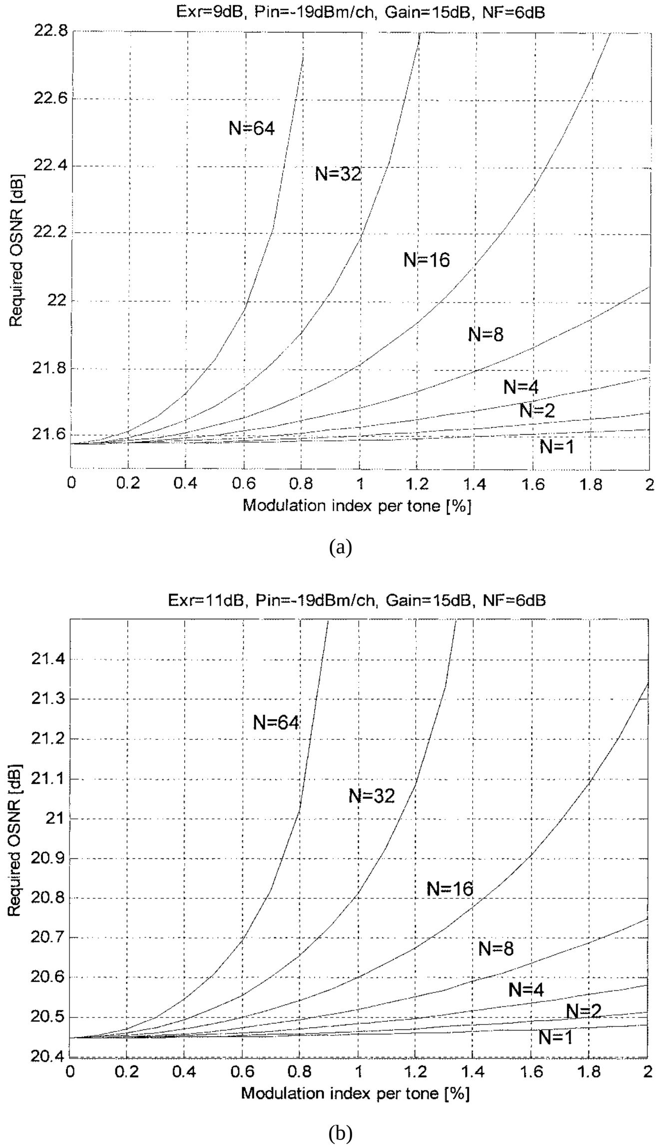


Fig. 3. Required OSNR for 10^{-16} BER in 10-Gb/s transmission link as a function of modulation index at (a) 9 dB of extinction ratio and (b) 11 dB of extinction ratio.

BER. Fig. 3(a) is the plot for the case of 9-dB extinction ratio. Increasing the modulation index requires larger OSNR to maintain the same BER. We can apply up to 0.75% of modulation index per tone if we accept 1-dB OSNR penalty when 64 tones with the same modulation index are imposed on the signal. When the extinction ratio is 11 dB in Fig. 3(b), up to 0.9% of modulation index per tone is available. It may be noted that the OSNR penalty depends on the extinction ratio of the optical signal as well as the modulation indexes.

We can also see the Q-penalty instead of OSNR penalty for evaluation. In Fig. 2, we got 10^{-22} BER with no modulation tone and about 10^{-18} BER with eight modulation tones with 2% modulation index. Here, the corresponding Q factor is 9.74 and 8.76, respectively. Therefore, in this case, the Q-penalty due to the modulation tones is $20 \cdot \log_{10}(9.74/8.76) = 0.92$ dB. We plot the relationship between the Q-penalty and OSNR penalty in dB-scale with variation of modulation index and number of

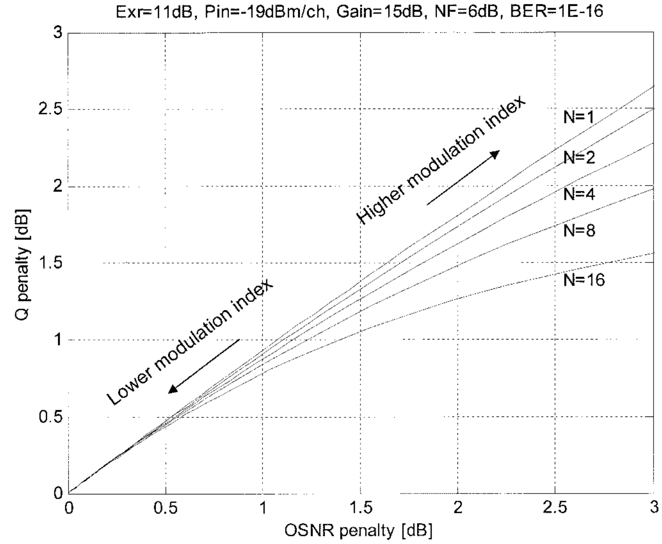


Fig. 4. Q-penalty versus OSNR penalty at 10^{-16} BER caused by N intensity modulation tones with modulation index variation. The range of modulation index was from 0% to 39% when $N = 1$, and from 0% to 1.4% when $N = 16$. With negligible modulation tones, the Q penalty and OSNR penalty have almost the same value in dB scale, because the Q is the square root of OSNR. However, as the amplitude of the modulation is increasing, the OSNR of the signal is getting smaller so that more OSNR improvement is required to compensate the BER degradation..

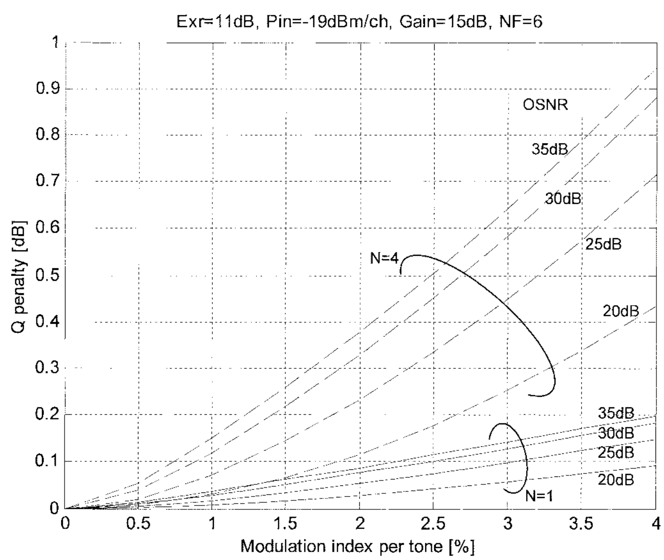


Fig. 5. Q-penalty increment variation at different OSNR. The group of solid lines represents the case of single tone, while the dashed lines shows when four tones are simultaneously imposed. As the OSNR is higher, the penalty due to the low-frequency intensity modulation appears more clearly.

tones in Fig. 4. For each curve, the range of modulation index is different among them: from 0% to 39% when $N = 1$, and from 0% to 1.4% when $N = 16$. As you can see, the Q-penalty has almost the same value of OSNR penalty with low modulation index because the Q factor is proportional to the square root of OSNR. However, as we increase the modulation index, the degradation of the line signal becomes larger so that OSNR penalty is getting larger than Q-penalty.

We plot the Q-penalty due to the low-frequency modulation in Fig. 5 when a single tone (solid line) and four tones with

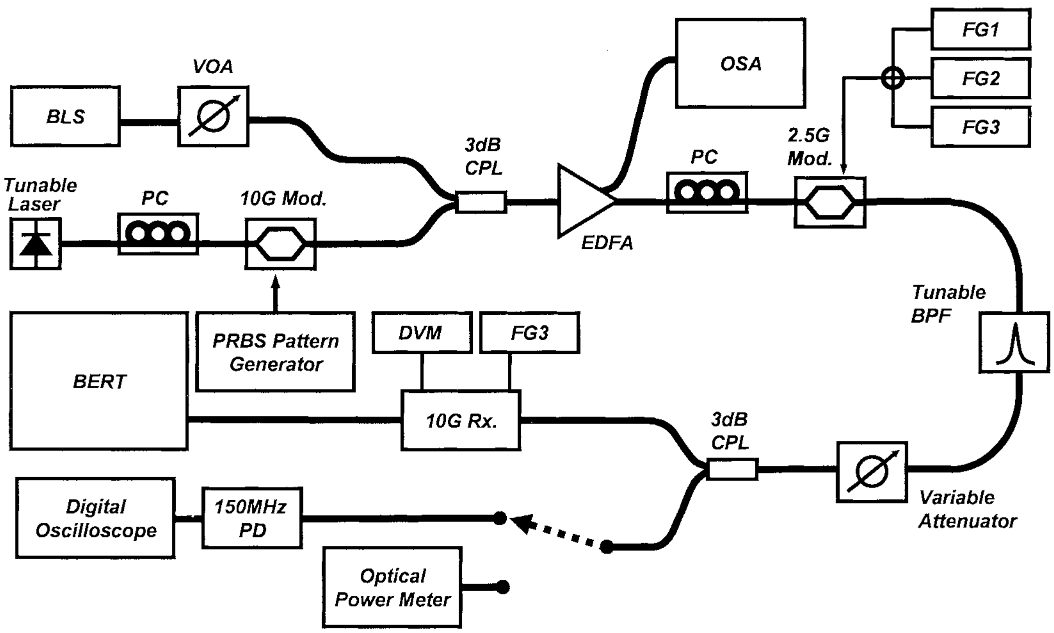


Fig. 6. Experiment setup to measure Q-penalty.

the same modulation index (dashed line) are imposed on the line-signal respectively. The input power to the preamplifier was −19 dBm for all cases. Preamplifier gain and NF are 15 and 6 dB, respectively. As we decrease the OSNR, Q penalty decreases at a given modulation index. It can be explained by increase of the background noise as we decrease the OSNR, i.e., the high background noise conceals the intensity modulation induced penalty.

IV. Q-Penalty Measurement

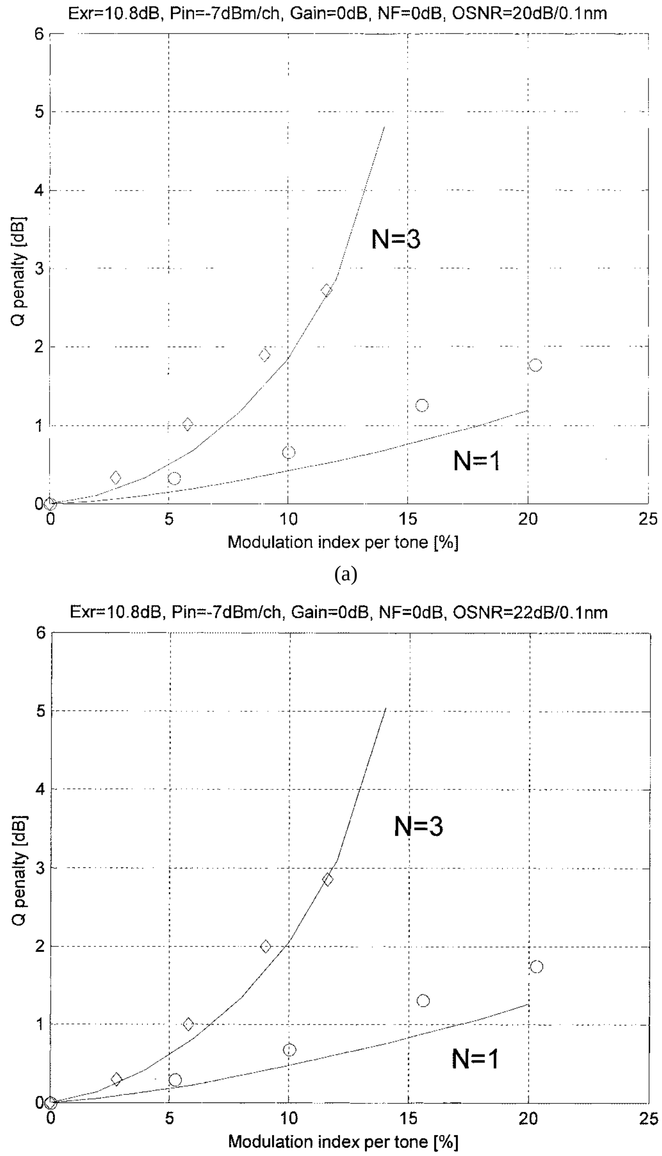
We measured experimentally the Q-penalty induced by the low-frequency intensity modulation with the setup shown in Fig. 6. We used an additional external modulator to impose the low-frequency intensity modulation signals on the extenally modulated 10-Gb/s line signal. The modulation tones of 1, 3, and 5 MHz from three function generators (FG1∼FG3) were applied to the additional external modualtor. We used a broadband light source and a 3-dB coupler to adjust the OSNR of the line signal. An erbium-doped fiber amplifier (EDFA) is used to compensate the losses by two modulators and two 3-dB couplers. We monitored the OSNR from a tap at this EDFA output. We used a tunable filter with 0.3 nm of full-width at half-maximum to emulate an optical channel drop. The optical signal was parallelly monitored by a low-bandwidth PD connected to a digital oscilloscope as well as by an optical power meter to measure the modulation index of pilot tones. By sweeping the decision threshold level of the receiver, we obtained the V-curves and calculated the Q factor [7]. As shown in Fig. 7, measured Q-penalty shows good agreement with the estimated Q-penalty by the analytic model for the small-indexed modulation. As the modulation index increases, the error between the simulation result and measured data is increased because the assumption $m \ll 1$ becomes inaccurate.

V. Discussion and Conclusion

Based on the previous study, we can estimate the performance degradation of the optical transmission link affected by the low-frequency intensity modulation. The low-frequency intensity modulation affects the performance of the digital transmission system when the modulation frequency is higher than the low-frequency cutoff of the optical receiver. Moreover, the modulation tones are accumulated when the signal passes each amplifier that has acoustooptic gain equalizer [3]. Assuming that the number of modulation tones is two in each amplifier and the span length is 80 km, then the total accumulated tone becomes 64 after 32 amplifiers. If we want to transmit the optical signal up to 2500 km with less than 1-dB OSNR penalty incurred as a result of such accumulated intensity modulation, the all-modulation tones added in each amplifier should have a modulation index of less than 0.8% per tone.

The upper limit of the modulation index when our model falls apart depends on the number of modulation tones. For a single modulation tone, the model holds with about 10% of modulation index. However, it decreases sharply as we increase the number of modulation tones, since there exists a coherent addition of the multiple modulation tones at a particular time.

In conclusion, we have examined the effect of low-frequency intensity modulation imposed on the line signal of an optical transmission link. The penalty caused by a single tone or multitone can be expressed as OSNR or Q penalty, which is the function of the extinction ratio, OSNR, modulation index, and number of tones. With this proposed model, the penalty due to the intensity modulation can be estimated analytically for design of the optical transmission systems that have optical performance monitors or active acoustic devices derived by low-frequency pilot tones. The proposed analytic model agreed well with the experimental results.



(a)

(b)

Fig. 7. Comparison between the measured Q-penalty (discrete points) and the estimated data (solid line) calculated using the proposed receiver model at 20- and 22-dB OSNR per 0.1 nm.

Appendix

A. BER Calculation

Q factor Q_i can be simplified as a form of

$$Q_i = \alpha_i \left(1 + \beta_i m \cos\omega_m t\right), \quad i = 0, 1 \tag{A1}$$

where α_i and β_i are obtained by comparing (A1) and (15). In the case of no ASE modulation, we can take them from (9) and (10) instead of (15).

The noise distribution in the receiver can be approximated as Gaussian distribution. Therefore, we can use the well-known equation for the BER

$$\begin{aligned}\text{BER} &= \frac{1}{4}\text{erfc}\left(\frac{Q_1}{\sqrt{2}}\right) + \frac{1}{4}\text{erfc}\left(\frac{Q_0}{\sqrt{2}}\right)\\ &\cong \frac{1}{2}\frac{1}{\sqrt{2\pi}Q_1}\exp\left(-\frac{Q_1^2}{2}\right) + \frac{1}{2}\frac{1}{\sqrt{2\pi}Q_0}\exp\left(-\frac{Q_0^2}{2}\right).\end{aligned} \tag{A2}$$

Under the assumption of $\beta_i m \ll 1$, the exponential notation in (A2) can be approximated to the modified Bessel function of the first kind, as we can see from (A5). We eliminate the square term for small m and use $(1 + x) \cong (1 - x)$ so that

$$\begin{aligned}&\frac{1}{\sqrt{2\pi}Q_i}\exp\left(-\frac{Q_i^2}{2}\right)\\ &= \frac{1}{\sqrt{2\pi}\alpha_i\left(1 + \beta_i m\cos\omega_m t\right)}\exp\left(-\frac{\alpha_i^2}{2}\left(1 + \beta_i m \cos\omega_m t\right)^2\right)\\ &\cong \frac{1 - \beta_i m\cos\omega_m t}{\sqrt{2\pi}\alpha_i}\exp\left(-\frac{\alpha_i^2}{2}\left(1 + 2\beta_i m\cos\omega_m t\right)\right)\\ &\cong \frac{1}{\sqrt{2\pi}\alpha_i}\exp\left(-\frac{\alpha_i^2}{2}\right)\cdot\left(1 - \beta_i m\cos\omega_m t\right)\\ &\quad\cdot\left[\text{I}_0\left(\alpha_i^2\beta_i m\right) + 2\sum_{n=1}^{\infty}(-1)^n\,\text{I}_n\left(\alpha_i^2\beta_i m\right)\cos n\omega_m t\right].\end{aligned} \tag{A3}$$

Here, we did use the following relation:

$$e^{iz\cos\theta} = \sum_{k=-\infty}^{\infty} i^k \text{J}_k(z) e^{ik\theta} \text{ and } \text{I}_k(z) = i^{-k}\text{J}_k(iz) \tag{A4}$$

where $J_k(z)$ represents the Bessel function of the first kind of order k.

Averaging over a multiple of time interval $T = 2\pi/\omega_m$, we can cancel out the all sinusoidal terms in (A3). Therefore, (A2) will become

$$\begin{aligned}\text{BER} =& \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}\alpha_1}\exp\left(-\frac{\alpha_1^2}{2}\right)\text{I}_0\left(\alpha_1^2\beta_1 m\right)\right.\\ &\left.+\frac{1}{\sqrt{2\pi}\alpha_0}\exp\left(-\frac{\alpha_0^2}{2}\right)\text{I}_0\left(\alpha_0^2\beta_0 m\right)\right].\end{aligned} \tag{A5}$$

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Jin-Serk Baik received the B.S. and M.S. degrees in electrical engineering from Korea Advanced Institute of Science and Technology, Daejeon, in 1999 and 2001, respectively, where he is currently pursuing the Ph.D. degree in optical communication systems.

From 2001 to 2002, he was with Novera Optics Korea, Inc., as a part-time Researcher to develop an automatic gain-controlled optical amplifier using acoustooptic tunable filters. His research interests include dynamic optical amplifiers and a low-cost high-performance optical access network based on WDM-PON.

Kun-Youl Park received the B.S. degree from Pusan National University, Pusan, Korea, in 1999 and the M.S. degree from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, in 2001, both in electrical engineering. He is currently pursuing the Ph.D. degree at KAIST.

From 2001 to 2002, he was with Novera Optics Korea, Inc., as a part-time Researcher and contributed to developing a dynamic control algorithm of an automatic gain-controlled optical amplifier and its modeling. His research interests include the modeling of optical amplifiers and injection-locked lasers using external incoherent light.

Tae-Won Oh received the B.S. degree from Kyungpook National University, Taegu, Korea, in 1995 and the M.S. and Ph.D. degrees from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, in 1997 and 2002, respectively, all in electrical engineering.

Currently, he is doing postdoctoral work with the Information and Electronics Research Institute, KAIST. His major research interests include lightwave transmission systems, optical amplifiers, and integrated-optic devices.

Chang-Hee Lee (M'97) received the B.S. degree from Hemyang University, Seoul, Korea, in 1983 and the M.S. and Ph.D. degrees from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, in 1985 and 1989, respectively, all in electrical engineering.

He did postdoctoral work with Bellcore from 1989 to 1990. From 1989 to 1997, he was with the Electronics and Telecommunications Research Institute, Daejeon. In 1997, he joined the Department of Electrical Engineering and Computer Science, KAIST, as an Associate Professor. Currently, he is a Professor in the Micro & Lightwave Group, Department of Electrical Engineering and Computer Science, KAIST and the Representative Director of Novera Optics Korea Inc. His major research interests include all-optical networks, lightwave transmission systems, lasers, and optoelectronic devices.

Dr. Lee is a Member of the Optical Society of America (OSA).